

Inference at * 1 1
of proof for Lemma equiv_rel_functionality_wrt_iff:

1. $T : \text{Type}$
2. $T' : \text{Type}$
3. $E : T \rightarrow T \rightarrow \mathbb{P}$
4. $E' : T' \rightarrow T' \rightarrow \mathbb{P}$
5. $T = T'$
6. $\forall x, y:T. E(x,y) \iff E'(x,y)$

$$\begin{aligned}
&\vdash ((\forall a:T. E(a,a)) \\
&\quad \& (\forall a, b:T. E(a,b) \Rightarrow E(b,a)) \\
&\quad \& (\forall a, b, c:T. E(a,b) \Rightarrow E(b,c) \Rightarrow E(a,c))) \\
&\iff ((\forall a:T'. E'(a,a)) \\
&\quad \& (\forall a, b:T'. E'(a,b) \Rightarrow E'(b,a)) \\
&\quad \& (\forall a, b, c:T'. E'(a,b) \Rightarrow E'(b,c) \Rightarrow E'(a,c))) \\
&\text{by InteriorProof ((RW (HigherC (HypC 6)) 0) \\
&\text{CollapseTHENA ((Auto.aux (first_nat 1:n} \\
&\quad) ((first_nat 1:n),(first_nat 3:n)) (first_tok :t) inil.term)))}.
\end{aligned}$$

1:

$$\begin{aligned}
&\vdash ((\forall a:T. E'(a,a)) \\
&\quad \& (\forall a, b:T. E'(a,b) \Rightarrow E'(b,a)) \\
&\quad \& (\forall a, b, c:T. E'(a,b) \Rightarrow E'(b,c) \Rightarrow E'(a,c))) \\
&\iff ((\forall a:T'. E'(a,a)) \\
&\quad \& (\forall a, b:T'. E'(a,b) \Rightarrow E'(b,a)) \\
&\quad \& (\forall a, b, c:T'. E'(a,b) \Rightarrow E'(b,c) \Rightarrow E'(a,c))) \\
&\quad .
\end{aligned}$$